internal diffusion transport of sugar within the capillary porous colloidal body.

The data obtained are necessary for calculation of mass exchange electroextractors which could be used in the food and other industries.

NOTATION

μ, root of characteristic equation; $\tan \alpha$, tangent of averaging straight line; D, sucrose diffusion coefficient in capillary porous body with electric field applied; β, mass transfer coefficient; R, reduced particle radius; d_e, equivalent diameter of interparticle channel; f, particle cross-sectional area; P, particle cross section perimeter; γ, specific weight of particles; q, specific extraction chamber load; w, extraction liquid velocity in channel; V, volume of extraction liquid; F_1, total cross-sectional area of interparticle channels; F_0, cross-sectional area of extraction chamber.

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KINETICS OF COMBUSTION OF POLYDISPERSED COKE DUST

Yu. M. Goldobin   UDC 621.1.016:536.46

A method for calculating combustion processes in self-similar systems of polydispersed solid fuel is developed, based on the kinetic equation for the particle distribution function over radius.

The study of combustion of polydispersed fuel is beset by difficulties related to the nonlinearity of the problem and the presence of the various temperature, combustion rate, and reagent concentration distributions, etc. In fuel combustion polydispersity of the particles is usually considered either by calculating the amount of fuel which remains unburned by a particular time, a quantity dependent only on the initial particle size distribution [1, 2], or by dividing the particle size spectrum into a number of fractions and performing calculations for each fraction [3]. These methods do not provide simple analytical expressions for calculation of combustion characteristics, and require use of numerical methods.

In [4], in a study of kinetics of mass exchange of a system of droplets with the surrounding medium polydispersity was considered by introducing a kinetic equation for the distribution function f(r, τ) of particles over radius. The present study will analyze solutions of this equation in detail, including self-similar systems, and generalize the result to a system with arbitrary initial distribution $f_0(r)$. It will be shown that the exact form of $f_0(r)$ affects only the initial stage of the process. The possibility of such an approach has been tested in a number of studies of heat–mass exchange of polydispersed systems with the surrounding medium [5-7], and generalized to some extent in [8].

In the present study a kinetic equation for the particle distribution function over radius will be used to solve the problems of combustion of polydispersed coke dust under adiabatic conditions. It will be assumed that the original dust consists of spherical particles of various size, that internal reactions are absent, that combustion occurs only in a primary reaction with formation of carbon dioxide, that the reaction rate is first order with respect to oxygen, and that the combustion is quasisteady-state and self-similar. The dust is injected into a medium of diluted oxidizer with initial temperature above the ignition temperature and ignition time will be neglected.

The kinetic equation for the particle distribution function over radius $f(r, \tau)$ has the form [4, 5]

$$\frac{\partial f(r, \tau)}{\partial \tau} + \frac{\partial}{\partial r} [f(r, \tau) W(r, \tau)] = 0. \tag{1}$$

For the function $f(r, \tau)$ the conditions

$$dN(\tau) = N_0 f(r, \tau)\, dr; \quad f(r, 0) = f_0(r); \quad \int_0^\infty f_0(r)\, dr = 1 \tag{2}$$

are satisfied. The combustion rate of an isolated particle was determined in [9]

$$W(r, \tau) = \frac{dr}{d\tau} = -\frac{1}{\frac{1}{k} + \frac{1}{\alpha_d}} \frac{\beta C_{ox}}{\rho_p}, \tag{3}$$

where $k = k_0 \exp[-E/RT_p]$; $\alpha_d = Nu_d D/2r$.

When Eq. (3) is used, the distribution function cannot be determined by the method of separation of Fourier variables, so convenient for self-similar systems [4], but it can be found for the limiting cases of combustion.

1. Diffusion Region. From Eq. (3) we find

$$w(r, \tau) = -\frac{\beta C_{ox}}{\rho_p} \frac{Nu_d D}{2r} = -\frac{\beta C_{ox_0}}{\rho_p} \frac{Nu_d D_0}{2r} \frac{T_g}{273}, \tag{4}$$

where $C_{ox} = C_{ox_0}\left(\frac{273}{T_g}\right)$; $D = D_0\left(\frac{T_g}{273}\right)^2$.

The empirical expression $Nu_d = 2 + br^m$ is valid for the diffusion Nusselt number over a wide range of gas velocities relative to the particles. The distribution function can be found from Eq. (1) in quadratures, but to find the moments of various order it is necessary to use numerical methods, so we will consider only the cases of large and small Reynolds numbers Re.

For large values of $Re(Nu_d = br^m)$

$$W(r, \tau) = \Omega(r) \omega(\tau) = \frac{1}{r^{1-m}}\left(\frac{\beta b C_{ox} D}{2\rho_p}\right), \tag{5}$$

where

$$\Omega(r) = \frac{1}{r^{1-m}}; \quad \omega(\tau) = -\frac{\beta b C_{ox} D}{2\rho_p}.$$

Then from Eq. (1) with consideration of Eq. (5), using the method of separation of Fourier variables we obtain a general solution of the equation [10]

$$f(r, \tau) = \sum_{i=1}^l A_i r^{1-m} \exp\left[-\frac{a_i}{2-m} r^{2-m}\right] \exp\left[a_i \int_0^\tau \omega(\tau)\, d\tau\right], \tag{6}$$

where a_i and A_i are constants, and from the condition ($\tau = 0$) the initial particle-size

distribution

$$f(r, 0) = f_0(r) = \sum_{i=1}^{l} A_i r^{1-m} \exp\left[-\frac{a_i}{2-m} r^{2-m}\right]. \tag{7}$$

It is evident from Eq. (7) that $f_0(r)$, usually calculated with the Rosin–Rammler formula, can be approximated by a set of spectral functions which define the choice of separation constants a_i and thus is related to combustion kinetics.

Following [4, 5], we calculate the moment of order s:

$$\langle r^s \rangle = \frac{N_0}{N(\tau)} \int_0^\infty r^s f(r, \tau)\, dr = \frac{N_0}{N(\tau)} \sum_{i=1}^{l} A_i a_i^{\frac{s-m+2}{2-m}} (2-m)^{\frac{s}{2-m}} \times$$
$$\times \Gamma\left(\frac{s+2-m}{2-m}\right) \exp\left[a_i \int_0^\tau \omega(\tau)\, d\tau\right]. \tag{8}$$

Considering that beginning with time τ_0 a unique regular regime sets in [4, 5], we obtain

$$\langle r^s \rangle = \frac{N_0}{N(\tau)} A a^{\frac{s-m+2}{2-m}} (2-m)^{\frac{s}{2-m}} \Gamma\left(\frac{s+2-m}{2-m}\right) \exp\left[a \int_{\tau_0}^\tau \omega(\tau)\, d\tau\right]. \tag{9}$$

From the normalization condition (s = 0) we have

$$N(\tau) = \frac{A}{a} N_0 \exp\left[a \int_{\tau_0}^\tau \omega(\tau)\, d\tau\right]. \tag{10}$$

We not introduce the fraction of the dust mass y which is still unburned at time τ:

$$y(\tau) = \frac{M_f(\tau)}{M_{f,0}} = \frac{1}{\overline{r_0^3}} \int_0^\infty r^s f(r, \tau)\, dr = \frac{\langle r^s \rangle N(\tau)}{\overline{r_0^3} N_0}. \tag{11}$$

From Eqs. (9)-(11) at s = 1 we obtain

$$a = \frac{(2-m)\Gamma^{2-m}\left(\frac{3-m}{2-m}\right)}{(\overline{r_0})^{2-m}}; \quad \langle r^s \rangle = \overline{r_0^s} = \frac{\overline{r_0^s}\, \Gamma\left(\frac{s+2-m}{2-m}\right)}{\Gamma^s\left(\frac{3-m}{2-m}\right)}. \tag{12}$$

The fraction of unburned fuel mass is determined from Eq. (11) with consideration of Eqs. (9), (10), (12)

$$y = \frac{A}{a} \exp\left[a \int_{\tau_0}^\tau \omega(\tau)\, d\tau\right]. \tag{13}$$

We obtain a different equation for y from Eq. (13)

$$\frac{dy}{d\tau} = a\omega(\tau)\, y. \tag{14}$$

The oxygen concentration and diffusion coefficient appearing in $\omega(\tau)$, Eq. (5), depend on the gas temperature T_g, therefore to solve Eq. (14) it is necessary to express T_g as a function of y.

We write the thermal balance equation for the system, neglecting heat expanded in heating the particles:

$$(M_g c_g + M_{ox} c_{ox} + M_{cp} c_r) \frac{dT_g}{d\tau} = -Q_R \frac{dM_f}{d\tau}, \tag{15}$$

from the solution of which for initial conditions y(0) = 1, $T_g = T_{g0}$ we obtain

$$T_g = T_{g0} + \frac{Q_R}{c_r} \ln \left[1 + \mu \frac{c_r}{c_g} (1 - y) \right], \tag{16}$$

where $\mu = \frac{M f_0}{M_g}$; $c_r = \frac{n_f}{n_{ox}} c_{ox} + \frac{n_f}{n_{cp}} c_r$.

Substituting T_g in Eq. (5) and expressing the oxygen concentration in the gases in terms of the fuel concentration, we obtain a nonlinear differential equation for $y(\tau)$, the solution of which cannot be expressed in elementary functions, requiring use of numerical methods. Linearization of the natural logarithm in Eq. (16) with respect to the parameter μ allows us to obtain a solution in final form. In this case we have

$$T_g \approx T_{g0} + \vartheta^* (1 - y), \tag{17}$$

where $\vartheta^* = \mu \frac{Q_R}{c_g}$.

Calculation of the fraction of unburned fuel y from the exact Eq. (16) and the approximate Eq. (17) shows that their results differ by no more than 4% for the entire range of μ possible.

From Eqs. (14), (5), (17) we obtain

$$\frac{dy}{d\tau} = - ky^2 (1 + k^* - k^* y), \tag{18}$$

where $k = a^* T_{g0}$; $a^* = \frac{abC f_0 D_0}{2 \rho_p 273}$; $k^* = \frac{\vartheta^*}{T_{g0}}$.

Equation (18) can be solved for τ:

$$\tau = \frac{1}{k(1 + k^*)} \left[\frac{k^*}{1 + k^*} \ln \left| \frac{1 + k^*(1 - y)}{y} \right| + \frac{1 - y}{y} \right]. \tag{19}$$

For low values of $Re(Nu_d = 2)$ the combustion rate can be determined from Eq. (4)

$$W(r, \tau) = \Omega(r) \omega(\tau) = - \frac{1}{r} \frac{\beta C_{ox} D_0 T_g}{\rho_p 273}, \tag{20}$$

where $\Omega(r) = \frac{1}{r}$; $\omega(\tau) = - \frac{\beta C_{ox_0} D_0 T_g}{\rho_p 273}$.

Then from Eq. (1) we obtain the particle distirbution function over radius

$$f(r, \tau) = \sum_{i=1}^{l} A_i r \exp \left[- \frac{a_i r^2}{2} \right] \exp \left[a_i \int_0^\tau \omega(\tau) d\tau \right]. \tag{21}$$

Having performed the above calculations, we obtain the self-similar parameters

$$a = \frac{2 \Gamma^2 \left(\frac{3}{2} \right)}{(\bar{r}_0)^2}; \quad \langle r^s \rangle = \bar{r}_0^s = \frac{\bar{r}_0^s \Gamma \left(\frac{s + 2}{2} \right)}{\Gamma^s \left(\frac{3}{2} \right)} \tag{22}$$

and an equation for $y(\tau)$ in the form of Eq. (18) with parameters

$$k = a^* T_{g0}; \quad a^* = \frac{a C f_0 D_0}{\rho_p 273}; \quad k^* = \frac{\vartheta^*}{T_{g0}}.$$

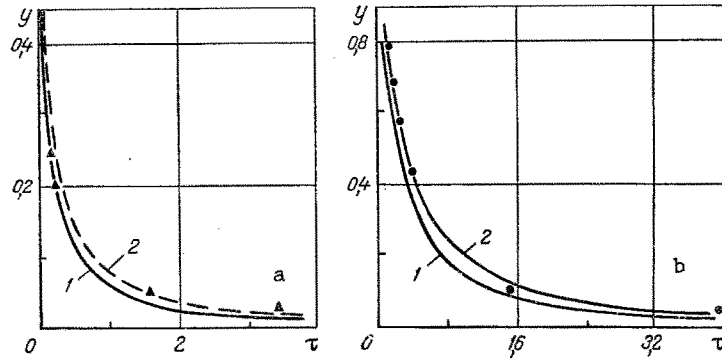2. **Kinetic Region.** The reaction rate is determined from Eq. (3)

Fig. 1. Combustion of polydispersed coke dust: a) 1, calculation with Eq. (19); 2, calculation by method of [11]; b) 1, calculation with Eq. (19) for $\overline{r}_0 = 30$ μm; 2, same for $\overline{r}_0 = 40$ μm; points, experiment [11]; τ, sec.

$$W(r, \tau) = \Omega(r)\,\omega(\tau) = -\frac{\beta C_{\text{ox}}}{\rho_p}k_0 \exp\left[-\frac{E}{RT_p}\right],\tag{23}$$

where

$$\Omega(r) = 1; \quad \omega(\tau) = -\frac{\beta C_{\text{ox}}}{\rho_p}k_0 \exp\left[-\frac{E}{RT_p}\right].$$

The particle distribution function over radius has the form

$$f(r, \tau) = \sum_{i=1}^{l} A_i \exp\left[-a_i r\right] \exp\left[a_i \int_0^\tau \omega(\tau)\,d\tau\right].\tag{24}$$

Calculation of moments of order s yields

$$\langle r^s \rangle = \frac{N_0}{N(\tau)} \sum_{i=1}^{l} A_i a_i^{-(s+1)} \Gamma(s+1) \exp\left[a_i \int_0^\tau \omega(\tau)\,d\tau\right].\tag{25}$$

Repeating all the previous transforms, we obtain

$$a = \frac{1}{\overline{r}_0}; \quad \langle r^s \rangle = \overline{r_0^s} = \overline{r}_0^s\tag{26}$$

and Eqs. (13), (14), but with different parameters. To relate the gas temperature and the fraction of unburned fuel we use the same Eqs. (15), (16), (17), as for combustion in the diffusion region. Substituting in Eq. (14) the value of $\omega(\tau)$ from Eq. (20) and considering that for combustion in the kinetic region the particle temperature is close to the gas temperature, Eq. (17), we find

$$\frac{dy}{d\tau} = -a^* \frac{\exp\left[-\dfrac{E}{R\left[T_{g_0} + \vartheta^*(1-y)\right]}\right]}{T_{g_0} + \vartheta^*(1-y)} y^2,\tag{27}$$

where $a^* = \dfrac{a C_{g_0} 273}{\rho_p} k_0$.

Writing Eq. (27) in dimensionless form, we write the solution as

$$\tau = \frac{1}{\bar{a}^*} \int_1^y \frac{(\theta_m - \theta^* y) \exp\left(\dfrac{1}{\theta_m - \theta^* y}\right)}{y^2} \, dy, \tag{28}$$

where

$$\bar{a}^* = \frac{a C_{fo} \theta_0 k_0}{\rho_p}; \quad \theta_0 = \frac{R273}{E}; \quad \theta^* = \frac{R\vartheta^*}{E};$$

$$\theta_m = \theta_{g0} + \theta^* = \frac{R(T_{g0} + \vartheta^*)}{E}; \quad \theta_{g0} = \frac{RT_{g0}}{E}.$$

The integral in Eq. (28) is calculated with the expression

$$\tau = \frac{1}{\bar{a}^*} \left\{ \frac{\theta_m - \theta^* y}{y} \exp\left(\frac{1}{\theta_m - \theta^* y}\right) - \theta_{g0} \exp\left(\frac{1}{\theta_{g0}}\right) + \right.$$

$$+ \frac{\theta^*}{\theta_m}(1 - \theta_m) \exp\left(\frac{1}{\theta_m}\right) \left\{ Ei\left(\frac{\theta^*}{\theta_m \theta_{g0}}\right) - Ei\left[\frac{\theta^* y}{\theta_m(\theta_m - \theta^* y)}\right]\right\} +$$

$$\left. + \theta^* \left[Ei\left(\frac{1}{\theta_{g0}}\right) - Ei\left(\frac{1}{\theta_m - \theta^* y}\right)\right]\right\}.$$

A comparison of calculated and experimental data [11] on combustion of anthracite culm dust with our calculations for combustion in the diffusion region are shown in Fig. 1a. All regime parameters were taken from [11], and the initial particle size distribution was calcu-lated after Eq. (3) with $\bar{d}_0$ = 44 μm. Also shown for comparison are experimental data on combustion of anthracite culm dust in a TP-70 boiler furnace [11] (see Fig. 1b). Samples of the medium for analysis were taken along the flame axis, so it can be assumed that in the horizontal portion of the flame conditions were adiabatic. Calculations were performed for an initial mean particle diameter obtained from the grain characteristics presented in [11]. Also shown for comparison are calculated data for a smaller particle size. The initial gas temperature was taken as T_{g0} = 1073°K, and the case of low Reynolds numbers Re (Nu_d = 2) was considered.

Calculations of coke burnup in the kinetic region with Eq. (29) show that over a time of 0.1-0.2 sec particles with initial size up to 10-15 μm burnup, which is confirmed by data on combustion of dust in a TP-230 boiler furnace [11].

NOTATION

r, particle radius: τ, time: $f(r, \tau)$, particle distribution function over radius; $W(r, \tau)$, combustion rate; $N(\tau)$ and N_0, current and total initial number of particles; k, reaction rate constant; k_0, preexponential factor; E, activation energy; R, ideal gas constant; T, temperature; α_d, mass exchange coefficient; D, diffusion coefficient; Nu_d, diffusion Nusselt number; β, ratio of molecular masses of carbon and oxygen; C, concentration; ρ, density; b, set of thermophysical parameters; m, exponent; Γ, gamma-function; M, mass; c, specific heat of combustion of fuel; θ_m, θ_{go}, θ_0, θ^*, initial fuel concentration in inert gas; n, molecular mass; μ, dimensionless temperatures: total maximum, initial gas, initial flow with particles, and maximum increase upon total burnup of fuel; E_i, integral exponential function. Subscripts: d, diffusion; ox, oxygen; O, initial value; f, fuel; g, inert gas; cp, combustion products; r, reduced; p, particle.

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ORTHOTROPIC PLATE WITH INCLUSION HEATED BY A HEAT SOURCE

Yu. M. Kolyano, I. R. Tatchin,   UDC 536.24
and E. G. Ivanik

The article presents solutions of steady problems of heat conduction for an ortho-tropic plate with foreign inclusion of arbitrary and small thickness.

We consider an orthotropic plate with thickness 2δ with an inclusion in the form of a strip of width $2h$. We represent the thermophysical characteristics of the system under examination in the form

$$p(x) = p^{(1)} + (p^{(0)} - p^{(1)}) N(x),\tag{1}$$

where $p^{(0)}$ and $p^{(1)}$ are the characteristics of the inclusion and of the base material, respectively, $N(x) = S_+(x+h) - S_-(x-h)$, $S_\pm(\xi)$ are asymmetric unique functions [1]. Heat exchange with the environment is effected through the surfaces $z = \pm\delta$ according to Newton's law. For determining the temperature we have the equation [2]

$$\frac{\partial}{\partial x}\left[\lambda_x(x)\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[\lambda_y(x)\frac{\partial T}{\partial y}\right] - \frac{\alpha_z(x)}{\delta}T = -w.\tag{2}$$

Heating of a Plate by a Linear Heat Source. We assume that an infinite orthotropic plate with an inclusion in the form of a strip $2h$ wide is heated by a linear heat source of intensity q, situated at the center of the inclusion. To determine the stationary temperature field, we have Eq. (2), where $w = \frac{q}{2\delta}\delta(x)\delta(y)$, and the boundary conditions

$$\lim_{|x|\to\infty} T = 0.\tag{3}$$

Taking (1) into account and using the formula

$$(\varphi\psi)' = \varphi'\psi + \varphi\psi' \mp [\varphi][\psi]\delta_\pm(x - x_1),\tag{4}$$